Difference between Area-based and Cordon-based Congestion Pricing: 
Investigation by Trip-chain-based Network Equilibrium Model with Non-additive Path Costs

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ABSTRACT

The modeling of the effects of congestion pricing has become a focus of attention and several researchers have analyzed the cordon-based congestion pricing problem. However, few have attempted to investigate area-based congestion pricing because it is considered difficult to analyze precisely. Cordon-based pricing can be expressed easily in a traffic assignment procedure by adding a charge to the inbound links of a cordon area, but the impedance of area-based pricing cannot be expressed using a link-based formulation. The objectives of this study are to propose a sound network model for evaluating area-based pricing and compare the effects of area-based pricing and cordon-based pricing. First, we point out that an exact treatment of area-based congestion pricing requires consideration of the non-additive trip-chain-based path cost. Such consideration appears very complicated, but we formulate a simple trip-chain-based network equilibrium model with non-additive path cost and present a convex minimization problem that is equivalent to the model. This formulation enables us to evaluate the effect of area-based congestion charging more exactly than with the traditional trip-based model, even for a large network. Finally, we apply the model to a real urban area (Okinawa, Japan) and compare area-based pricing with cordon-based pricing. In this example application, the optimal toll level for area-based pricing is found to be higher than that for cordon-based pricing.
INTRODUCTION

The evaluation of congestion pricing policy has been attracting a great deal of attention recently. Methods of congestion pricing can be classified as point-based, time-based, distance-based, cordon-based and area-based pricing. Several researchers have analyzed the cordon-based congestion pricing problem (1-8) in investigations of cordon location, charging levels, and social benefits. In a cordon-based scheme, drivers have to pay to enter a designated zone. Cordon-based pricing has certain practical advantages: cordon tolls are transparent, as drivers know the charge beforehand, they are reliable and easy to understand and use, they are relatively simple to implement and the technology has already been tested and is available for wide use. Further, cordon-based pricing is easy to implement not only in the practical sense of transport policy but also in the transport modeling sense. The impedance of cordon-based pricing can be expressed easily by adding a pricing charge to the inbound link of the cordon area in the transportation network model. In other words, cordon-based pricing is a link-additive cost and can be expressed using a standard traffic assignment model.

In comparison, let’s consider the case of an area-based congestion charging scheme or area licensing scheme. London adopted this scheme in February 2003. In area-based congestion pricing, drivers pay to enter a designated area or to drive in the area, but can drive freely within the area for the whole day. Area-based pricing can be more difficult to implement in practice than cordon-based pricing, especially if the charging area is large. This is because all cars within the pricing area have to be monitored; with cordon-based pricing only cars entering the cordon have to be checked. Despite this implementation difficulty, area-based pricing might be better in a social welfare sense than cordon-based pricing because it can be seen as closer to first-best pricing. In a first-best pricing scheme, a toll that is equal to the difference between the marginal social cost and the marginal private cost is charged on each link in the network. This means there is a growing need for a means of precisely evaluating area-based pricing. However, an exact evaluation of area-based pricing is not easy because no link-based expression of the system is possible, unlike with cordon-based pricing. Whether a driver within the area should be charged or not depends on whether or not s/he has previously paid the toll. In this case, the impedance of area-based pricing depends not only on location but also on the driver’s route history for the day. Area-based pricing is not link-additive but history-dependent. If we do not treat these dependencies properly, pricing evaluations will be biased and may underestimate pricing revenue or overestimate traffic reductions.

The dependencies of area-based pricing can be fully considered using the concept of a drivers’ one-day trip-chain. With area-based congestion pricing, drivers have to pay at most once per trip-chain. Area-based pricing can be regarded as a non-additive trip-chain cost. Drivers will make decisions to change behavior in response to congestion pricing according to the trip-chain cost. Therefore we need a proper model to express the cost structure.

The objectives of this paper are

[1] to develop a sound trip-chain-based network equilibrium model with non-additive trip-chain cost aiming at exact evaluation of area-based congestion pricing, and

[2] to apply the model to a real urban area and investigate the difference between area-based and cordon-based congestion pricing.

An appropriate model is developed by extending the trip-chain-based network equilibrium model described in our previous research (11). This model will be a lower-level
problem of several kind of MPEC (Mathematical Programming with Equilibrium Constraints) in future work, such as optimal toll pricing problem.

The paper is organized as follows. First, we recall the basic model for trip-based elastic-demand network equilibrium and its trip-chain-based counterpart. Then we propose a novel model for trip-chain-based network equilibrium with non-additive cost, describing the formulation and algorithm. Next we apply the model to a real urban area and compare the effect of area-based and cordon-based pricing in view of social surplus and pricing revenue. Finally the results are summarized with a discussion and conclusion.

**FORMULATION OF BASIC MODELS**

**Basic Beckmann Model**

First we look at the basic model proposed by Beckmann (12) in order to facilitate understanding of our extended model. This model is formulated as the well-known convex minimization problem below (see e.g. 13).

\[
\min Z_1(f, q) = \sum_a \int_0^{x_a} t_a(\omega) d\omega - \sum_{r,s} \int_0^{q_{rs}} D_{rs}^{-1}(\omega) d\omega \tag{1}
\]

subject to

\[
\sum_k f_{rs}^k = q_{rs}, \quad \forall r, s \tag{2}
\]

\[
x_a = \sum_{r,s,k} \delta_{a,k} f_{rs}^k, \quad \forall a \tag{3}
\]

\[
x_a \geq 0, f_{rs}^k \geq 0, q_{rs} \geq 0 \tag{4}
\]

where

- \(x_a\): traffic flow on link \(a\);
- \(t_a(x_a)\): Travel cost on link \(a\), which is a function of the traffic flow on the link, and is assumed to be separable and an increasing function of traffic flow \(x_a\);
- \(q_{rs}\): OD (Origin-Destination) travel demand from zone \(r\) to zone \(s\);
- \(D_{rs}^{-1}(\cdot)\): Inverse of the demand function associated with OD pair \(rs\);
- \(f_{rs}^k\): Traffic flow from zone \(r\) to zone \(s\) on route \(k\);
- \(\delta_{a,k}^{rs}\): 1 if link \(a\) is on route \(k\) between OD pair \(rs\) and 0 otherwise;

and \(f\) denotes the vector of \(f_{rs}^k\) and \(q\) the vector of \(q_{rs}\).

Travel demand in the model is determined by a *trip-based* demand function \(D_{rs}(.).\) The first-order conditions for this problem are user equilibrium conditions while OD demand is determined by a trip-based demand function. This model is used extensively in the literature to analyze the congestion pricing problem (3-10).

**Trip-chain-based Network Equilibrium Model**
The demand function given in subsection above is defined on a trip basis. In this section, we redefine the trip-chain based demand function. The trip-chain based model is expressed using the notation proposed by Maruyama and Harata (11). We adopt general trip-chain modeling including piston-type trip-chains (i.e., journeys with a single destination and two symmetric trips). Trip chain \( n \) is denoted by a set \( \{n_0, n_1, n_2, \ldots \} \) that contains origin node \( n_0 \) and transmitting nodes \( n_1, n_2, \ldots \) in the order visited in the trip-chain. We assume the set of possible trip chains is predetermined. Then we consider the following mathematical program.

\[
\min Z_2(f, q, h) = \sum_n \int_0^{s_n} t_a(\omega) \, d\omega - \sum_n \int_0^{k_n} D_n^{-1}(\omega) \, d\omega
\]

subject to

\[
q_{rs} = \sum_n h_n^{rs} \eta_n^{rs}, \quad \forall r, s
\]

\[
\sum_k f_k^{rs} = q_{rs}, \quad \forall r, s
\]

\[
x_a = \sum_{r, s, k} \delta_{a, k} f_k^{rs}, \quad \forall a
\]

\[
x_a \geq 0, f_k^{rs} \geq 0, q_{rs} \geq 0
\]

where

\( h_n \): travel flow on trip chain \( n \);

\( \eta_n^{rs} \): 1 if trip chain \( n \) contains OD pair \( rs \) (that is \( n = \{\ldots, r, s, \ldots\} \)) and 0 otherwise;

and \( h \) denotes the vector of \( h_n \). Then the Lagrangian of the mathematical program can be written as

\[
L = Z_2 + \sum_{rs} \mu_{rs} \left( q_{rs} - \sum_n h_n^{rs} \eta_n^{rs} \right) + \sum_{rs} \gamma_{rs} \left( q_{rs} - \sum_k f_k^{rs} \right)
\]

here \( \mu_{rs}, \gamma_{rs} \) are Lagrange multipliers associated with (6) and (7), respectively. Hence,

\[
\frac{\partial L}{\partial q_{rs}} = \mu_{rs} + \gamma_{rs}
\]

\[
\frac{\partial L}{\partial h_n} = -D_n^{-1}(h_n) - \sum_{rs} \mu_{rs} \eta_n^{rs}
\]

\[
\frac{\partial L}{\partial f_k^{rs}} = c_k^{rs} - \gamma_{rs}.
\]

Then, if we assume \( q_{rs} > 0 \), the first-order conditions are

\[
\frac{\partial L}{\partial q_{rs}} = 0, \frac{\partial L}{\partial h_n} = 0, \frac{\partial L}{\partial h_n} \geq 0, \frac{\partial L}{\partial f_k^{rs}} = 0, \frac{\partial L}{\partial f_k^{rs}} \geq 0.
\]

Therefore we have

\[
f_k^{rs}(c_k^{rs} - c_r) = 0, c_k^{rs} - c_r \geq 0, \quad \forall r, s, k
\]

\[
h_n(c_n - D_n^{-1}(h_n)) = 0, c_n - D_n^{-1}(h_n) \geq 0, \quad \forall n
\]

\[
c_n = \sum_{rs} c_{rs} \eta_n^{rs}, \quad \forall n.
\]

Here \( c_{rs} (\equiv -\mu_{rs} = \gamma_{rs}) \) is the minimum cost between OD pair \( rs \) and we define \( c_n \) as the total travel cost of trip chain \( n \). Equation (15) gives the user equilibrium conditions between OD.
pair \( rs \). Equation (16) is relationship between trip-chain-based travel demand and trip-chain-based travel cost. This proves the equivalency between (5)-(9) and the trip-chain-based network equilibrium model incorporating general trip chaining behavior.

There are two differences between our model given as eqns. (5)-(9) and the original Beckmann model of eqns. (1)-(4). First, the second term of the objective function is based on the inverse of the trip-chain based demand function and second eqn. (6) is added to transform the trip-chain-based flow to trip-based OD flow. Note that in the above model we assume that users visit a node once at most in a trip chain. To relax this assumption, we have to consider cyclic trip chaining; a traveler may visit a node more than one time. We can easily incorporate this behavior if we redefine \( \eta_{rs} \) as \( i \) if the trip chain \( n \) from contains OD pair \( rs \) \( i \) times and 0 otherwise.

In this model, travel cost is determined by link-additive performance. Hence, it can be used for cordon-based congestion pricing. Note that using this model we can evaluate homeward trip pricing in a cordon-based system, which may be difficult to evaluate using a trip-based model. Maruyama and Harata (11) investigated cordon pricing using a simpler version of the above trip-chain-based model in a simple network. As mentioned before, any attempt to evaluate area-based congestion pricing requires the construction of another model.

**TRIP-CHAIN-BASED NETWORK EQUILIBRIUM MODEL WITH NON-ADDITIVE COST**

**Formulation**

We propose here a trip-chain-based network model with non-additive cost along trip-chain-based path. We introduce the notion of a trip-chain-based path. We consider each trip-chain \( n \) to be connected by a set of trip-chain-based paths through the network. If each OD pair \( rs \) in a trip-chain has \( N_{rs} \) trip-based paths, then the total number of trip-chain-based paths is \( \prod_{rs} N_{rs} \).

We define a set of trip-chain-based path as \( M \). As we have pointed before, area-based congestion pricing can be expressed in trip-chain-based paths. The toll level paid by drivers depends on whether or not a particular trip-chain-based path contains the charging area.

If there is only one charging area, the situation can be explained rather simply. The set of trip-chain-based paths \( M \) is divided into two sets, \( M_0 \) and \( M_1 \). The two sets are mutually exclusive (i.e., \( M_0 \cap M_1 = \emptyset, M_0 \cup M_1 = M \)). \( M_0 \) contains trip-chain-based paths that need no charging. \( M_1 \) contains trip-chain-based paths that are charged. Then the generalized cost of each trip-chain-based path can be expressed as follows:

\[
C^m_n = \begin{cases} 
\sum_a \delta_{a,n}^m t_a(x_a), & \text{if} \ m \in M_0 \\
\sum_a \delta_{a,n}^m t_a(x_a) + \tau_1, & \text{if} \ m \in M_1
\end{cases}
\]

(18)

where \( \delta_{a,n}^m \) is redefined as 1 if link \( a \) is on trip-chain-based path \( m \) in trip-chain \( n \) and 0 otherwise, and \( \tau_1 \) is the toll level of area-based pricing converted into time units according to the value of time. Thus, the trip-chain-based path cost function has a non-additive type cost function; this function cannot be decomposed into link-based terms.
Next let’s consider a situation where there is more than one charging area, such as double area-based pricing. Note that the toll imposed on drivers who go through two charging areas may not be the sum of the two area-based tolls, because pricing may be discounted. In order to express such a situation in general terms, we define a notation for the charging pattern along trip-chain-based paths. The charging pattern classifies a trip-chain-based path according to the total amount of toll charges along the path. Here we denote each charging pattern as \( p \). Generally speaking if there are \( n \) charging areas then the total number of charging patterns \( P \) is \( 2^n \). For example, if there is only one charged area, then the charging pattern will be either \( p = 0 \) (paying no toll) or \( p = 1 \) (paying the toll) as discussed in the above paragraph. Then a set of trip-chain-based paths \( M \) is divided into \( P \) mutually exclusive sets \( \{ M_p \} \) (i.e., \( M_{p1} \cap M_{p2} = \emptyset \), \( \forall p1, p2 \), \( \bigcup_p M_p = M \)). Here \( M_p \) contains a trip-chain-based path with charging pattern \( p \). Then generalized cost of each trip-chain-based path \( m \) in trip-chain \( n \) can be expressed as follows:

\[
 c_n^m = \sum_a \delta_{a,n}^m t_a(x_a) + \tau_p, \quad \text{if } m \in M_p
\]  

(19)

where \( \tau_p \) is the toll level of charging pattern \( p \), converted into time units according to the value of time and we define \( \tau_0 = 0 \), and \( M_0 \) means the set of trip-chain-based paths that have no charge.

The next question is then how to model such non-additive costs for trip-chain-based paths. There have been several investigations of non-additive path cost models for trip-based models (14-17). In some cases, the models have been formulated as convex problems (16, 17). We follow this idea and extend it to the trip-chain based model.

Consider the following problem:

\[
 \min Z_3(g, h) = \sum_n \int_0^{h_n} t_a(\omega) d\omega + \sum_{n,m} \tau_p g_n^m - \sum_n \int_0^{h_n} D_n^{-1}(\omega) d\omega
\]  

(20)

subject to

\[
 h_n = \sum_m g_n^m, \quad \forall n,
\]  

(21)

\[
 x_a = \sum_{m,n} \delta_{a,n}^m g_n^m, \quad \forall a,
\]  

(22)

\[
 x_a \geq 0, h_n \geq 0, g_n^m \geq 0,
\]  

(23)

where \( g_n^m \) is the travel flow on trip-chain-based path \( m \) in trip-chain \( n \) and \( g \) denotes the vector of \( g_n^m \).

Then the Lagrangian of the mathematical program can be written as

\[
 L = Z_3 + \sum_n \mu_n \left\{ h_n - \sum_m g_n^m \right\}
\]  

(24)

here \( \mu_n \) are redefined as Lagrange multipliers associated with (21). Hence

\[
 \frac{\partial L}{\partial h_n} = D_n^{-1}(h_n) - \mu_n
\]  

(25)

\[
 \frac{\partial L}{\partial g_n^m} = \sum_a \delta_{a,n}^m t_a(x_a) + \tau_p - \mu_n, \quad \forall n, \forall m \in M_p.
\]  

(26)

Then we have following the first-order conditions.

\[
 g_n^m(c_n^m - c_n) = 0, c_n^m - c_n \geq 0, \quad \forall m, n
\]  

(27)
where \( c_n(\equiv -\mu_n) \) is the minimum trip-chain-based path cost in trip-chain \( n \) and \( c_n^m \) is defined by (19). In this case, the extended user equilibrium condition defined in the trip-chain-based path of (27) and (28) is clear. Equation (27) means that if the trip-chain-based path flow is zero (i.e. \( g^m_n = 0 \)) then the trip-chain-based travel cost on this trip-chain-based path, \( c_n^m \), must be greater than or equal to the minimum trip-chain-based path cost in the trip-chain, \( c_n \); otherwise, if the trip-chain-based path flow is positive, the trip-chain-based travel cost must be equal to the minimum trip-chain-based path cost. Therefore, if the trip-chain-based flow satisfies these equations, no driver will be better off by unilaterally changing his/her trip-chain-based path. Note that in the case of the trip-chain-based network equilibrium model with additive path costs shown in (5)-(9), the user equilibrium condition can be stated by OD-based formula (15), but in the case of a non-additive path cost model we can only state the equilibrium condition using trip-chain based formula.

We now look into the uniqueness of the solutions to the problem (20)–(23) and the resulting revenue for given toll charges. Due to the strict convexity of the objective function in \( h \) and \( x \), the trip-chain flow and link flow solutions are unique. But the solution to trip-chain-based path flow is not uniquely determined because the second-term of the objective function (20) is linear in \( g \). Nevertheless, the revenue collected from toll charges is uniquely defined. The reason for this is similar to that for the trip-based model (17). For a given convex program, there is a unique optimal objective function value. The value of the first and third terms is unique, so the second term, total revenue, is also uniquely determined.

Solution Method

As with the traditional Beckmann model, we can use the method of partial linearization (or the double stage algorithm) to solve the problem. The point is that, although trip-chain-path flow is used in the formulation, the problem can be solved without enumeration of the trip-chain-path flow, meaning the model is applicable to large-scale networks.

In developing the algorithm, we define

\[
d_n^p = \sum_{m \in M_n} g^m_n, \quad \forall n, p,
\]

as total travel flow in trip-chain \( n \), which will be imposed tolls with charging pattern \( p \). Using this notation, objective function (20) can be expressed without trip-chain-based path flow as follows:

\[
\min Z_3(d, h) = \sum_a \int_0^{x_a} t_a(\omega) d\omega + \sum_{n, p} \tau_p d_n^p - \sum_n \int_0^{h_n} D_n^{-1}(\omega) d\omega
\]

where \( d \) denotes the vector of \( d_n^p \). An overview of the algorithm is given below.

**Step 0: Initialization.** Find a set of feasible flows \( X^{(1)} = \{x^{(1)}, d^{(1)}, h^{(1)}\} \). Set \( l = 1 \).

**Step 1: Travel-time update.** Calculate \( t_a = t_a(x^{(l)}_a), \quad \forall a \).

**Step 2: Direction finding.**

Solving partial linear approximation problem to obtain auxiliary solution \( Y = \{x^{sub}, d^{sub}, h^{sub}\} \).

**Step 3: Move-size determination.** Find the \( \alpha \) that solves
Step 4: Flow update. Set $X^{(l+1)} = X^{(l)} + \alpha(Y - X^{(l)})$.

Step 5: Convergence test. If convergence is not achieved, set $l := l+1$ and return to Step 1. Otherwise, terminate.

The objective function of the partial linear approximation problem in Step 2 is shown below.

$$\min \hat{Z}(g,h) = \sum_{m,n} c_n^m q_n^m - \sum_n \int_0^{h_n} D_n^{-1}(\omega) d\omega$$

Thus, finding the optimal solution to partial linearization problem (31) requires assignment of the demand $D_n(c_n)$ to the trip-chain-based path with the smallest cost, provided that $c_n^m \leq D_n^{-1}(h_n)$, and no flow should be assigned to any path if $c_n^m > D_n^{-1}(h_n)$. This allocation procedure requires finding the shortest trip-chain-based path for each trip-chain. Because the trip-chain-based path cost defined in eqn. (19) is not link-additive and depends on the charging pattern $p$, a slight modification of the standard assignment procedure is needed. To explain this, Step 2 is summarized in more detail below.

Step 2.1: For each charging pattern $p$, compute the shortest path between OD pairs based on link travel time $\{t_a\}$. If link $a$ cannot be used in a charging pattern, then the link cost should be set to $t_a = \infty$ or a sufficiently large value. This provides the shortest travel time $c_n^p$ between OD pairs $rs$ for each charging pattern $p$.

Step 2.2: From $\{c_n^p\}$, compute the shortest travel time for each trip chain $\tilde{c}_n^p$ for each toll pattern $p$ by the following method:

$$\tilde{c}_n^p = \sum_{rs} c_n^p \eta_n^rs, \quad \forall n, p.$$  

Recall that $\eta_n^rs$ is 1 if trip chain $n$ contains OD pair $rs$ and 0 otherwise. Then the generalized trip-chain cost for each charging pattern $p$ is

$$c_n^p = \tilde{c}_n^p + \tau_p, \quad \forall n, p.$$  

Then, for each trip-chain $n$, search for the charging pattern $p^*_n$ that gives the smallest generalized cost among $\{c_n^p\}$.

$$c_n = \min_p \{c_n^p\}, \quad p^*_n = \arg \min_p \{c_n^p\}$$  

Step 2.3: If $c_n \leq D_n^{-1}(h_n)$, set $d_n^p = h_n = D_n(c_n)$ and $d_n^p = 0, \forall p \neq p^*_n$.

If $c_n > D_n^{-1}(h_n)$, set $d_n^p = h_n = 0, \forall p$.

This procedure gives auxiliary trip-chain-based flows $\{h_{sub}\}, \{d_{sub}\}$.

Step 2.4: Assign $\{d_{sub}\}$ to the minimum trip-chain-based path for each charging pattern $p$. It is efficient to make the assignment by OD-based flow to the minimum-travel-time path between each OD pair for each charging pattern $p$ calculated in Step 2.1, using the following transformation:

$$q_n^{rs} = \sum_n d_n^p \eta_n^rs, \quad \forall r, s, p.$$
where $q_{rs}^p$ is OD travel flow from $r$ to $s$ by charging pattern $p$. This yields a link-flow pattern $\{x_{u}^{ab}\}$.

It should be noted that the direction selected in Step 2 is a descent direction of original problem and the algorithm does converge to the desired solution. See Patriksson (18) for a general proof. In Step 3 of the algorithm, we have to evaluate the value of the objective function. If we use (30) instead of (20) we can easily and efficiently evaluate the value. Note that we have to calculate the shortest travel time between OD pairs $P$ times at each iteration, which may cause additional computational effort than for the normal additive path cost model.

Social Benefit and Revenue

The basic calculated measures for this model are summarized below. The social surplus $SS$ (or social welfare value) of the model can be calculated as

$$SS = \sum_n \int_0^{h_n} D_n^1 (\omega) d\omega - \sum_u x_a t_a (x_a), \quad (36)$$

and the congestion pricing revenue $PR$ is

$$PR = \sum_{n,p} \tau_p d_n^p. \quad (37)$$

Finally, the user benefit $UB$ is

$$UB = SS - PR = \sum_n \int_0^{h_n} D_n^{-1} (\omega) d\omega - \sum_a x_a t_a (x_a) - \sum_{n,p} \tau_p d_n^p. \quad (38)$$

As in the case of the trip-based model, it is easily proved that the marginal cost pricing on every link gives the maximum social surplus ($SS$) in this trip-chain-based model. We use some of these measures for comparison of area-based and cordon-based pricing.

EXAMPLE OF APPLICATION TO REAL URBAN AREA

In order to demonstrate the effectiveness and applicability of the proposed model, we present an example of its application to a real urban area. We describe the study area and the data used in the next subsection, then a policy simulation using area-based and cordon-based congestion pricing follows in the next subsection.

Study Area, Data and Settings

We apply our model to Okinawa prefecture in Japan. Okinawa is located at the southernmost tip of Japan and has a population of nearly 1.4 million. We use data from the 1999 Road Traffic Census Survey. The traffic analysis zones are shown in FIGURE 1 and the arterial road network analyzed is shown in FIGURE 2. The total number of traffic analysis zones is 67, the number of links is 12,308 and the number of nodes is 5,462.
FIGURE 1 Traffic Analysis Zone in Okinawa Area
The link performance function takes the following form:

\[ t_a(x_a) = t^0_a \left( 1 + 0.48 \left( \frac{x_a}{C_a} \right)^{2.82} \right), \]  

where \( t^0_a \) is free-flow travel time and \( C_a \) is link capacity. We set the value of time at 60 Japanese yen/min, or approximately 0.5 US$/min. These parameters are based on standard values for Japan (19). We use following demand function:

\[ D_n(c_n) = D^0_n \exp \left( \rho \left( 1.0 - \frac{c_n}{c^0_n} \right) \right), \]  

where \( c_n \) and \( c^0_n \) are the equilibrium and free-flow trip-chain-based travel costs, respectively, on trip chain \( n \), and \( D^0_n \) is the corresponding potential demand. The demand elasticity with respect to the travel cost is given by \(-\rho c_n / c^0_n\) where \( \rho \) is regarded as a dimensionless demand elasticity parameter. There has been discussion about how to set the demand elasticity parameter (for example, in 10), and we choose \( \rho = 0.5 \) in our application. In these models with a direct demand function, drivers may choose not to make the trip-chain when
the charge applies or to use an alternative route to avoid it. These decisions are based on trip-chain based travel cost. Trip-chains that are cancelled will include trip-chains made by other modes, altered trip-chain patterns that avoid charges, or trip-chains that are cancelled altogether. A convergence criterion based on the change in link flow
$$\max \left\{ \frac{|x_a^{(t+1)} - x_a^{(t)}|}{x_a^{(t)}} \right\} < 0.1$$
is adopted. This criterion leads to convergence after around 100 iterations and the average excess cost (see e.g. 20) is around 0.01 (min) in our application. This rather mild criterion is set so as to compute many toll level scenarios.

FIGURE 3 shows the hypothetical charged area set up for our policy simulation. The charged area corresponds to the area inside the ring road and covers about 10 km². We use travel data for a whole day and assume that pricing is imposed evenly over the whole day. Making this assumption allows for analysis of the difference between area-based and cordon-based pricing with all available trip-chain data. If we focus on specific time periods, then the number of trip-chains that are completed in that time period will be restricted and the effect of trip-chaining will not be analyzed effectively.

![FIGURE 3 Central Area of Okinawa and Charged Area](image)

We use the master data from the Road Traffic Census Survey of 1999 (consisting of individual answer sheets in the form of a questionnaire). Each individual sample in the master data includes an expansion factor that can be used to amplify the values to represent the total population. We regard this expansion factor as the potential demand for trip-chain flow. The total number of trip data samples is 20,331 and the number of trip-chains is 6,997, so the average number of trips is 2.91 (= 20,331/6,997). Note that the number of OD pairs is 4,761 (67*67), so the memory required for this example is of the same order as for conventional trip-based assignment. FIGURE 4 shows the distribution of number of trips in a trip-chain in this area. In the sample data, 57.5% of trip-chains consist of two trips forming piston-type trip-chains. We eliminate data with more than 8 trips in a trip-chain for simplicity. Trip-chains with 8 trips or fewer amount to 95.4% of the total trip-chain count in the sample data.
Note that the input data for travel demand in our model is not a trip-based OD matrix but rather trip-chain-based flows. Therefore, standard traffic assignment software cannot be used in the application of this model. The number of feasible trip-chains will be infinite in the real world, so we have to consider a method of managing the data. We propose a data format for the representation of trip-chain flows. To save space in computer memory, this has a special form. FIGURE 5 is an example of a data table for trip-chain information. The data format consists of two tables; a trip-chain master table and an OD master table. The OD master table lists the origin and destination pairs in order of trip in a trip-chain, and the trip-chain master table holds a pointer for each record indicating the position, in the OD master table list, of the trip origin along with the number of trips in the trip-chain. Using this data format, all trip-chains can be processed easily and the trip-chain-based cost can be computed efficiently.
Results and Discussion

FIGURE 6 shows the difference in social surplus between cordon-based and area-based congestion pricing in this application. The social surplus function has a concave shape in both area-based and cordon-based pricing. If a stricter convergence criterion were to be adopted, the function may exhibit a smoother shape. We can infer that the optimal cordon-based toll, that is the one that maximizes the social surplus, is around 250 Japanese yen and the optimal area-based toll is around 500 Japanese yen. The optimal toll for each scheme gives similar optimal value of social surplus. If the tolled level is below 300 yen for both schemes, cordon-based pricing gives a higher social surplus compared to area-based pricing. On the other hand, if both tolls are above 300 yen, area-based pricing gives a higher social surplus than cordon-based pricing.

FIGURE 7 depicts the revenues collected with cordon-based and area-based pricing. Over the range of pricing levels investigated here, revenue increases with rising tolls in both pricing schemes. Where tolls are below 600 yen, revenue from a cordon-toll scheme is slightly greater than from an area-based scheme, while for tolls exceeding 600 yen, revenue from area-based pricing is greater than that from cordon-based pricing.
FIGURE 6 Difference in Social Surplus between Cordon-based and Area-based Congestion Pricing
Note) JPY= Japanese yen

FIGURE 7 Difference in Pricing Revenue between Cordon-based and Area-based Congestion Pricing
Note) JPY= Japanese yen
These differences can be explained as follows. In the case of cordon-based pricing, drivers may be tolled several times a day and this may cause overloaded pricing compared to first-best pricing. This overload can cause significant behavioral changes in drivers and induce a welfare loss at higher pricing levels. The overloaded tolls collected with cordon-based pricing yield more revenue at lower pricing levels than area-based pricing, but changes in driver behavior may lead to revenues below those of area-based pricing when pricing is set at a higher level. Figure 8 shows an illustrative example of the difference. In this example, area-based pricing results in a single charge whereas cordon-based pricing is tolled twice on the same trip-chain. These results further demonstrate that the social surplus under cordon-based pricing is more sensitive to toll level than under area-based pricing in our example. This indicates the need for more careful consideration of pricing level when cordon-based pricing is used in practice rather than area-based pricing.

In addition to the work carried out here, an investigation of equity variations may be useful. Area-based pricing may be more equitable than cordon-based pricing because the former charges trip-chains that remain within the charged area while the latter does not. Another definition of the charged area (e.g. a larger area) will give another variation of social surplus and pricing revenue. Such investigations will be among our future work.

![Figure 8 Illustrative Example of Difference between Area-based and Cordon-based Congestion Pricing](image-url)
CONCLUSION

To the authors’ knowledge, this is the first attempt to investigate the exact effects of area-based congestion pricing in comparison with cordon-based pricing. The investigation made use of a novel approach using a trip-chain-based network equilibrium model with non-additive costs.

We noted initially that a precise treatment of area-based congestion charging would require us to consider the non-additive trip-chain-based path cost. This would appear to be very complex, but fortunately we can formulate a model based on a convex minimization problem. This model enables us to evaluate the effect of area-based congestion charging more exactly than with a traditional trip-based model, even for a large-scale network.

We applied the model to the Okinawa area and compared area-based and cordon-based pricing. For such applications to real urban areas, we propose a data format for representing the trip-chain flows that offers efficient management of the trip-chain information. In our example application, the optimal toll level for area-based pricing was found to be higher than that for cordon-based pricing. Naturally, this result may depend on the network structure and travel demand characteristics of trip-chains in the study area. However, the model can be applied to any urban area for which master data from an OD survey is accessible, so we will be able to investigate these effects in other areas. Future investigations of other areas will give us additional insight into the properties of area-based congestion pricing. That is, the major contribution of this research is the proposal of a basic and sound model for investigating the differences between area-based and cordon-based pricing.

One limitation of this research is that we do not explicitly consider changing patterns of trip-chaining that are caused by congestion charging. In reality, drivers can alter the transmitting node of a trip chain, and such behavioral changes are expressed implicitly by the direct demand function. Explicit treatment of the changes would require us to prepare alternative trip chains, and this may make the model more complicated. Despite its relative simplicity, we believe the analysis made possible using this basic model can provide valuable information.

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