Incorporating Trip Chaining Behavior in Network Equilibrium Analysis

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Abstract

The network equilibrium model is a useful tool for long-term transportation planning, and is one promising alternative to the traditional four-step travel forecasting model. However there remain some outstanding issues to be considered. One is that almost all kinds of the model adhere to the traditional trip-based approach, where trip chains made by users are treated as separate, independent entities in the analysis. In this research, the aim is to develop a simple and tractable model that overcomes this problem. One proposed model is based on piston-type trip chaining and another accommodates any other type of trip chaining and includes congestion phenomena. Our models have certain special features. First, they have been successfully formulated as convex minimization problems, so uniqueness and algorithm convergence are easily proven. Second, traveler behavior is based on theoretically sound random utility models, allowing the benefit of transportation projects to be calculated such that it is consistent with travel demand forecasting. Thirdly, we can calculate optimal road pricing even in large-scale networks. We examine the model using simple network examples with special attention to the effect of trip chaining behavior on the level of second-best toll. In a simple two-destination network, the second-best toll of the trip-based model is lower than that of trip-chain-based model, indicating one of the biases of the trip-based model.
INTRODUCTION

The network equilibrium model is a useful tool for long-term transportation planning. It has been greatly improved through the seminal work of Beckmann, et al. (1). Recently Boyce and Bar-Gera (2) have reviewed progress in combined network equilibrium models with emphasis on the implementation and application of multiclass models. Such models overcome some of the drawbacks of the traditional four-step travel forecasting model, and so offer a promising alternative. One important shortcoming of the traditional approach is inconsistency among steps. For example, the OD travel time output from a traffic assignment may not be the same as the travel time input to the mode choice model. The combined network equilibrium approach can overcome this problem on a theoretically sound and rigorous mathematical foundation (3). Another problem is lack of behavioral theory behind the traditional model. To deal with this problem, Oppenheim (4) has presented a Nested Logit based network equilibrium approach based on the well-known random utility maximization behavioral theory. Therefore the network equilibrium approach may be a promising candidate for urban travel forecasting models in the next era. Recently, several attempts have been made to apply models based on this approach to real urban areas (5-9).

Some issues, however, remain to be considered. An important one is that almost all kinds of the model adhere to the traditional trip-based approach, where the trip is the unit of analysis, and trip chains made by an individual are treated as separate, independent entities in the analysis. Such traditional trip-based approaches often fail to recognize the existence of linkage among trips. This is the problem we attempt to tackle in this research. Typical of the problems that arise with network equilibrium models adhering to the trip-based approach are the following:

Problem [1]
In a combined trip distribution and assignment model, the destination of one trip may change as the result of a certain transportation policy. However, such a destination change may not be reflected by a change in the origin of the following trip.

Problem [2]
In a combined modal split and assignment model, the travel mode may change as the result of a certain transportation policy. In reality, if travelers change the mode for one trip they will also change it for the following trip. However, this behavior is seldom expressed in a network equilibrium model.

In this research, the aim is to develop a simple and tractable model that overcomes these two problems. Several attempts have been made to deal with these problems (10-13). Of particular note is the tour-based system that has formed part of the Dutch National Transportation Model for many years (e.g. 14). However, Cascetta (15) has stated that the mathematical models proposed to simulate trip chains do not have a “standard” structure like those for trip demand models, and this is partly due to the greater complexity of the phenomena to be represented. The simulation method is one possible approach, and implementing trip chaining in a simulation is not a particularly hard task. However, simulations suffer certain drawbacks; in particular, the mathematical properties of the model are unknown or unclear. An activity-based approach might be introduced. Lam and Yin (16), for example, have proposed a variational inequality model for the dynamic user equilibrium of activity/route choices. However, their algorithm is a heuristic one.

The authors believe it is better to accept the limitations of convex optimization models in representing certain features of the transportation system in order to avoid the risks of ill-behaved problems and/or algorithms, as stated by Bar-Gera and Boyce (17, p.409). Models with unique solutions will be effective because they can provide a robust analysis framework especially suitable for travel forecasting in long-term strategic planning. Therefore, we propose a simple and mathematically rigid model for analyzing trip chaining behavior in a static network equilibrium setting.

Several models are presented in this paper. The first is a combined trip distribution and assignment model. This consists of a piston-type trip-chain (round trip) choice and route choice network equilibrium model. It is able to solve the above-mentioned Problem [1]. The second is a combined modal split and assignment model. It consists of mode choice and route choice equilibrium in a piston-type trip-chain framework, and is able to solve the above-mentioned Problem [2]. The final one is a general trip-chain choice model including chain journeys with more than one destination. We further show an extension that covers cyclic trip chains. These
three models are successfully formulated as convex optimization problems, offering the advantages of unique solutions and algorithms that are known to converge.

Using these models, we then demonstrate some interesting policy analysis. First, we use them to design optimal link tolls and system optimal traffic flows. It is well known that a so-called marginal cost toll can drive fixed demand user equilibrium to a system optimum in which total travel time is minimized. It will be shown in this analysis that the marginal cost toll pricing is also an optimal link toll if the objective function of the system optimum is a social surplus consisting of the sum of toll revenue and a user surplus that is consistent with the trip chaining choice model. Second, we analyze second-best road pricing in a simple network. Recently, several authors ([18]) have explored optimal cordon-based network congestion pricing problems. But these analyses use a trip-based approach and the results may include a bias. Travelers who decide to change their destination from the cordoned area to another uncharged area will also change the characteristics of the following trip. We investigate the effect of this on optimal cordon pricing with a numerical example.

**FORMULATION OF PISTON-TYPE TRIP CHAINING MODEL**

*Piston-Type Trip-Chain-Based Combined Trip Distribution and Assignment Model*

We first give the simplest possible model for ease of understanding. We present a piston-type trip-chain-based combined trip distribution and assignment model that overcomes Problem [1].

Let’s consider the following convex optimization problem:

\[
\min \sum_{a} \int_{0}^{\infty} t_{a}(\omega) d\omega + \frac{1}{\theta} \sum_{r,s} h_{rs} \ln h_{rs} / O_r
\]

subject to

\[
\sum_{s} h_{rs} = O_r, \quad \forall r
\]

\[
q_{rs} = h_{rs} + h_{sr}, \quad \forall r,s
\]

\[
\sum_{k} f_{rs}^{\prime \prime} = q_{rs}, \quad \forall r,s
\]

\[
x_{a} = \sum_{r,s,k} \delta_{a,k} f_{rs}^{\prime \prime}, \quad \forall a
\]

\[
x_{a} \geq 0, f_{rs}^{\prime \prime} \geq 0, q_{rs} \geq 0, h_{rs} \geq 0
\]

where

- \( O_r \): Travel demand from origin zone \( r \) (a given fixed value);
- \( q_{rs} \): OD travel demand from zone \( r \) to zone \( s \);
- \( h_{rs} \): Travel demand of piston-type trip chain \( rs \). We define the piston-type trip chain \( rs \) as originating from zone \( r \), passing through zone \( s \), and returning to zone \( r \);
- \( f_{rs}^{\prime \prime} \): Traffic flow from zone \( r \) to zone \( s \) on route \( k \);
- \( x_{a} \): Traffic flow on link \( a \);
- \( t_{a}(x_{a}) \): Travel time on link \( a \), which is a function of the traffic flow on the link, and is assumed to be separable and an increasing function of traffic flow \( x_{a} \);
- \( \delta_{a,k} \): 1 if link \( a \) is on route \( k \) between OD pair \( rs \), and 0 otherwise;
- \( \theta \): Dispersion coefficient (to be estimated) for the trip chain choice model.

Note that the constraints (2)-(6) are linear and define a convex set, while objective function (1) is strictly convex in \( x, h \). Therefore solutions of trip chain flow \( h \) and link flow \( x \) are unique and OD flow \( q \) is also unique due to (3).

The Lagrangian of the mathematical program can be written as
\[ L = Z + \sum_r \lambda_r \left( O_r - \sum_s h_{rs} \right) + \sum_s \mu_{rs} \left( q_{rs} - (h_{rs} + h_{sr}) \right) + \sum_s \gamma_{rs} \left( q_{rs} - \sum_k f_k^{rs} \right) \]  

(7)

Then we have

\[ \frac{\partial L}{\partial q_{rs}} = \mu_{rs} + \gamma_{rs}, \]  

(8)

\[ \frac{\partial L}{\partial h_{rs}} = \frac{1}{\theta} \left( 1 + \ln(h_{rs}) \right) - \lambda_r - \mu_{rs} - \mu_{sr}, \]  

(9)

\[ \frac{\partial L}{\partial f_k^{rs}} = c_k^{rs} - \gamma_{rs}, c_k^{rs} = \sum_a L_k(x_a) \delta_{a,k}, \]  

(10)

where \( c_k^{rs} \) is the cost of route \( k \) between OD pair \( rs \). If we assume \( h_{rs} > 0, q_{rs} > 0 \), then the first-order conditions are

\[ \frac{\partial L}{\partial q_{rs}} = 0, \frac{\partial L}{\partial h_{rs}} = 0, f_k^{rs} = 0, \frac{\partial L}{\partial f_k^{rs}} \geq 0. \]  

(11)

Therefore we have

\[ f_k^{rs} (c_k^{rs} - c_r) = 0, c_k^{rs} - c_r \geq 0, \]  

(12)

\[ h_{rs} = O_r \exp\left\{ -\theta (c_r + c_s) \right\} / \sum_s \exp\left\{ -\theta (c_r + c_s) \right\}. \]  

(13)

where we define \( c_{rs} = -\mu_{rs} - \gamma_{rs} \) as the minimum cost of routes between OD pair \( rs \). Equation (12) shows user equilibrium conditions for the outward journey of trip chain \( rs \) and the homeward journey of trip chain \( sr \). Equation (13) is a logit model of trip chain choice behavior, and this choice is based on round trip travel cost \( c_r + c_s \). Therefore, we see that mathematical problem (1)–(6) is equivalent to a piston-type trip-chain-based combined trip distribution and assignment model (FIGURE 1).

Compared with the traditional singly-constrained combined trip distribution and assignment model (19), the difference is the second term of objective function (1) and constraint (3). If we consider the attraction measure \( M_s \) that is associated with destination zone \( s \), the model can be easily extended to include the term \( -\sum_r M_r h_{rs} \) in objective function (1). This singly-constrained model replicates \( O_r \), but does not necessarily replicate \( D_s \), the travel demand to destination zone \( s \), which may be a given value. One possible solution to this problem is to calibrate the attraction measure, \( M_s \), so that it correctly matches the flow constraint on destination zones.

**Solution Method**

As with the traditional combined model, we can use the partial linearization method to reach a solution. We describe the method briefly below.

**Step 0: Initialization.** Find a set of feasible flows \( X^{(1)} = \{ x^{(1)}, q^{(1)}, h^{(1)} \} \). Set \( n=1 \).

**Step 1: Travel-time update.** Calculate \( t_a = t_a(x^{(1)}), \forall a \).

**Step 2: Direction finding.**

(a) Calculate the route with the shortest travel time between OD pairs based on \( \{ t_a \} \). Let \( c_{rs} \) denote the shortest travel time from \( r \) to \( s \).

(b) Determine the auxiliary trip chain flows \( \{ h^{sub} \} \) using (13), and auxiliary OD flows \( \{ q^{sub} \} \) using (3).

(c) Assign \( \{ q^{sub} \} \) to the minimum-travel-time path between \( r \) and \( s \). This yields a link-flow pattern \( \{ x^{sub} \} \).

**Step 3: Move-size determination.** Find the \( \alpha \) that solves

\[ \min \quad Z(X^{(\alpha)} + \alpha(Y - X^{(\alpha)})), \quad \text{s.t.} \quad 0 \leq \alpha \leq 1 \]

where \( Y = \{ x^{sub}, q^{sub}, h^{sub} \} \).

**Step 4: Flow update.** Set \( X^{(\alpha+1)} = X^{(\alpha)} + \alpha(Y - X^{(\alpha)}) \).
Step 5: Convergence test. If convergence is not achieved, set \( n = n + 1 \) and return to step 1. Otherwise, terminate.

This algorithm is a slight modification of existing ones, and offers the potential for solving large network problems efficiently. The method is proven to converge to unique equilibrium solutions. The recently proposed origin-based algorithm (17) will be a promising alternative to this algorithm in our future work.

Piston-Type Trip-Chain-Based Combined Modal Split and Assignment Model

Next we present an alternative model that overcomes Problem [2]. Here we have a given piston-type trip-chain travel flow.

Consider the following mathematical program.

\[
\min \ Z(x, q, h) = \sum_{m=0}^{\infty} \int_{\omega}^{\infty} t_m^* (\omega) d\omega + \frac{1}{\theta} \sum_{m,m'} h_m^{m'} \ln \frac{h_m^{m'}}{h_m}
\]  

subject to

\[
\sum_{m} h_m^{m} = h_r, \quad \forall r, s, m
\]  
\[
q_r^{m} = h_r^{m} + h_s^{m}, \quad \forall r, s, m
\]  
\[
\sum_{a} f_a^{m} = q_r^{m}, \quad \forall r, s, m
\]  
\[
x_a^{m} = \sum_{r,s} S_{r,s,a} f_a^{m}, \quad \forall m, a
\]  
\[
x_a^{m} \geq 0, f_a^{m} \geq 0, h_r^{m} \geq 0, h_s^{m} \geq 0
\]  

Here, notation \( m \) represents the mode index. As in the previous section, the first-order condition of the above problem gives user equilibrium condition for each mode and the mode choice logit model,

\[
h_m^{m} = h_r \exp \left\{ \frac{\theta (c_m^m + c_s^m)}{\zeta} \right\}
\]  

Thus we can prove the equivalency between this mathematical program (14)- (19) and the piston-type trip-chain-based combined modal split and assignment model. This program can be easily solved by a similar solution method described in the previous section.

We can extend these models in the direction of stochastic route choice models, multi-class models, combined trip-distribution modal-split assignment models, or even the generation of trip chaining, semi-dynamic models. For example, if we add the following term to objective function (1)

\[
\frac{1}{\zeta} \sum_{m,a} f_a^{m} \ln \frac{f_a^{m}}{q_r^{m}}
\]  

where \( \zeta \) is dispersion parameter of route choice, then we have a stochastic user equilibrium model with a Nested Logit-type trip-chain and route choice behavior. Furthermore, integration of residential choice in location equilibrium modeling might be an effective extension.

In the next section, we present another extension aimed at relaxing the limitation that the whole trip chain should be of the piston type. However, in application to real urban areas, the above piston-type modeling should prove sufficient and extremely valuable.

EXTENSION TO GENERAL TRIP CHAINING

We consider general trip-chain modeling. We assume that the origin travel demand from each zone is given and this travel demand is all in the form of trip chaining in a network consisting of one travel mode.

For the purpose of this section, trip chain \( c \) is denoted by a set \( c = \{c_0, c_1, c_2, \ldots, c_n, c_0\} \) that contains origin node \( c_0 \) and transmitting nodes \( c_1, c_2, \ldots, c_n \) in the order visited in the trip chain. We assume the choice set of possible trip chains is predetermined. Then we consider the following mathematical program.
\[ \min Z(\mathbf{x}, \mathbf{q}, \mathbf{h}) = \sum_c \int_a^b t_c(\omega) d\omega + \frac{1}{\theta} \sum_c h_{pc} \ln h_{pc} / O_p \] (22)

subject to

\[ \sum_c h_{pc} = O_p, ~ \forall p \] (23)
\[ q_{rs} = \sum_c h_{pc} \eta_{pc}^r, ~ \forall r, s \] (24)
\[ \sum_k f_k^r = q_{rs}, ~ \forall r, s \] (25)
\[ x_a = \sum_{r,s,k} \delta_{a,k} f_k^r, ~ \forall a \] (26)
\[ x_a \geq 0, f_k^r \geq 0, q_{rs} \geq 0, h_{pc} \geq 0 \] (27)

where \( h_{pc} \) is redefined as travel flow on trip chain \( c \) originating from node \( p \). \( \eta_{pc}^r \) is 1 if trip chain \( c \) from origin \( p \) contains OD pair \( rs \) (that is \( c = [\ldots, r, s, \ldots] \)) and 0 otherwise. Using this notation, the total travel cost of trip chain \( c \) from origin \( p \) can be written as follows:

\[ c_{pc} = \sum_r c_r \eta_{pc}^r, ~ \forall r, s \] (28)

Then the Lagrangian of the mathematical program can be written as

\[ L = Z + \sum_p \lambda_p \left( O_p - \sum_c h_{pc} \right) + \sum_r \mu_r \left( q_{rs} - \sum_c h_{pc} \eta_{pc}^r \right) + \sum_s \gamma_s \left( q_{rs} - \sum_k f_k^r \right) \] (29)

Hence

\[ \frac{\partial L}{\partial q_{rs}} = \mu_r + \gamma_r \] (30)
\[ \frac{\partial L}{\partial h_{pc}} = \frac{1}{\theta} \left( 1 + \ln(h_{pc} / O_p) \right) - \lambda_p - \sum_r \mu_r \eta_{pc}^r \] (31)
\[ \frac{\partial L}{\partial f_k^r} = c_k^r - \gamma_r \] (32)

Thus, if we assume \( h_{pc} > 0, q_{rs} > 0 \), then the first-order conditions are

\[ f_k^r (c_k^r - c_{rs}) = 0, c_k^r - c_{rs} \geq 0 \] (33)
\[ h_{pc} = O_p \sum_c \exp(-\theta c_{pc}) \] (34)

Here \( c_{rs} = \mu_r \) is the minimum cost between OD pair \( rs \) and we use (28). Equation (33) gives the user equilibrium conditions between OD pair \( rs \). Equation (34) is a logit model of trip-chain choice behavior, and this choice is based on total trip-chain travel cost. Therefore, we have proven the equivalency between (22)-(27) and the combined network equilibrium model incorporating general trip chaining behavior.

Note that in above model we assume that users visit a node once at most in a trip chain. To relax this assumption, we have to consider cyclic trip chaining; a traveler may visit a node more than one time. We can easily incorporate this behavior if we redefine \( \eta_{pc}^r \) as \( n \) if trip chain \( c \) from origin \( p \) contains OD pair \( rs \) \( n \) times and 0 otherwise.

A NUMERICAL EXAMPLE

Two-Destination Example

Consider the simple network (Network A) depicted in FIGURE 2, consisting of 1 origin, 2 destination nodes, and 4 links. The link performance function takes the following form:
The free-flow travel time and capacity of the links are given in TABLE 1. There is a travel demand of 2,000 (veh/h) from origin node 0. Travelers from origin node 0 choose a destination from among node 1 and node 2, travel there, and return to origin node 0. This trip-chain choice model is described by equation (13). We assume $\theta = 0.01$ (/min) and homogeneous travelers with a single value of time equal to 50 JPY/min.

Our theoretically consistent model can easily calculate optimal link tolls. In order to calculate optimal tolls, we modify the link performance function (35) to be the marginal cost function for every link. By making such settings, the following social surplus that is consistent with our model is maximized:

$$SS = \frac{1}{\theta} \sum_{a} Q_{a} \ln \sum_{r} \exp[-\theta(c_{ra} + c_{ar})] + \sum_{a} x_{a} p_{a},$$

(36)

where $p_{a}$ is the toll on link $a$. The first term of (36) is a user surplus that is consistent with our behavioral model and the second term is the total pricing revenue.

Initial equilibrium solutions are shown in TABLE 2 (in the no-toll equilibrium column). In this case, we assume piston type trip chaining, so the flow on link $(1,0)$ is equal to that on link $(0,1)$ and that on link $(2,0)$ is also equal to that on link $(0,2)$. TABLE 2 also shows optimal toll equilibrium solutions. The link tolls for links $(0,1)$ and $(1,0)$ are higher than those for links $(0,2)$ and $(2,0)$, respectively. Therefore some travelers modify their destination from node 1 to node 2, decreasing the flow on link $(0,1)$ and increasing it on link $(0,2)$. In addition the flow on link $(1,0)$ and link $(2,0)$ changes accordingly, because our model is trip-chain-based and the homeward trip also varies.

In this first best situation, every link is charged. However, this may be impractical in a real situation. Therefore, we investigate the second-best situation where only link $(1,0)$ can be charged. This type of second-best pricing has recently been examined by many researchers, but so far no one has analyzed it using trip-chain modeling.

We try several toll levels on link $(1,0)$, and FIGURE 3 shows the results. In this figure, the social surplus is normalized; the surplus at no-toll equilibrium is 0% and that at optimal toll equilibrium is 100%. The social surplus function has a concave shape with an optimal value. We can see that the second-best toll can also lead to first best optimal surplus at a toll level of around 1,500 to 1,600 (JPY) in this simple network. FIGURE 3 also shows the results of the usual trip-based model to indicate the different effect of trip chaining. The trip-based model is a singly constrained trip distribution and assignment model that considers only the outward trip while neglecting the homeward trip. We can see also that a second-best toll of around 1,100 (JPY) will lead to first best surplus with trip-based modeling.

The optimum second-best toll given by the trip-chain-based model is higher than that by the trip-based model. This is because congestion externality in homeward trips as well as in outward trips is considered in the trip-chain-based model. Thus, we might infer from this test network trial that the second-best toll calculated by the trip-based model is lower than the actual second-best toll. This inference can change according to network characteristics, so we consider another example in the next subsection.

Cordon Pricing Example

Consider the network (Network B) depicted in FIGURE 4, consisting of 4 nodes and 10 links. The input data for this network are given in TABLE 3. Other assumptions are the same as before. TABLE 4 compares the no-toll equilibrium solution with the optimal toll equilibrium solution.

In this network we try to simulate a cordon pricing policy, so links $(1,2)$, $(4,2)$, and $(3,2)$ are assumed to be charged at the same toll level in a second-best toll situation. FIGURE 5 shows the results of calculating the second-best toll with both trip-based and trip-chain-based models. We can see that, in this cordon pricing policy, the second-best toll level given by the trip-chain-based model is quite similar to that of the trip-based model. This inference may change according to network characteristics. However, our model can be applied to large-scale networks, so we can investigate these effects in any example network. Further comparisons will be investigated as part of our future research.
CONCLUSION

In this research, we propose some simple combined network equilibrium models that incorporate trip chaining behavior. These models overcome certain problems associated with trip-based models, such as neglecting changes in homeward trip routes according to certain transport policies.

One of our models is based on piston-type trip chaining, which is a minor extension of the traditional trip-based model. This allows us to make use of existing software and algorithms designed for the combined network equilibrium model to express piston-type trip chaining. Another model accommodates all other types of trip chaining. This is a general model, thus it might become the standard model for future extensions.

Our models have certain special features. First, they are successfully formulated as convex minimization problems, so problem uniqueness and algorithm convergence are easily proven. Second, traveler behavior is based on theoretically sound random utility models, allowing the benefit of transportation projects to be calculated such that it is consistent with travel demand forecasting. Thirdly, we can calculate optimal road pricing even in large-scale networks.

We have examined the models in simple network examples while paying particular attention to the effect of trip chaining behavior on the level of the second-best toll. In a simple two-destination network, the second-best toll given by the trip-based model was lower than that by the trip-chain-based model, illuminating one of the biases of trip-based models. On the other hand, in a cordon pricing network example, the difference between the results was quite small.

This research has some potential for future extension. Introducing the concept into transport-location equilibrium modeling and spatial computable general equilibrium modeling may be two useful extensions. There may be a large number of heuristic approaches to expressing trip chain behavior in these models, but the original contribution made by this research is to demonstrate that there is a simple and sound formulation by which these phenomena may be expressed using a convex optimization approach.

ACKNOWLEDGEMENT

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### TABLE 1 Input data for Test Network A

<table>
<thead>
<tr>
<th>Link</th>
<th>$t_0$: Free-flow travel time (min)</th>
<th>$C$: Capacity (vehicle/h)</th>
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<td>500</td>
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<tr>
<td>(1,0)</td>
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<td>500</td>
</tr>
<tr>
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<td>(2,0)</td>
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### TABLE 2 Equilibrium Solution for Test Network A

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<td>Link travel time (min)</td>
<td>Link flow (veh)</td>
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<td>(3,1)</td>
<td>10</td>
<td>1,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node</th>
<th>Travel demand from each node (veh/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trip-based</td>
</tr>
<tr>
<td>1</td>
<td>1,000</td>
</tr>
<tr>
<td>2</td>
<td>1,000</td>
</tr>
<tr>
<td>3</td>
<td>1,000</td>
</tr>
<tr>
<td>4</td>
<td>1,000</td>
</tr>
</tbody>
</table>
TABLE 4 Equilibrium Solution of Test Network B

<table>
<thead>
<tr>
<th>Link</th>
<th>No-toll equilibrium</th>
<th>Optimal toll equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Link flow (veh)</td>
<td>Link travel time (min)</td>
</tr>
<tr>
<td>(1,2)</td>
<td>651</td>
<td>7.15</td>
</tr>
<tr>
<td>(2,1)</td>
<td>651</td>
<td>7.15</td>
</tr>
<tr>
<td>(2,4)</td>
<td>651</td>
<td>7.15</td>
</tr>
<tr>
<td>(4,2)</td>
<td>651</td>
<td>7.15</td>
</tr>
<tr>
<td>(2,3)</td>
<td>349</td>
<td>5.177</td>
</tr>
<tr>
<td>(3,2)</td>
<td>349</td>
<td>5.177</td>
</tr>
<tr>
<td>(3,4)</td>
<td>329</td>
<td>10.018</td>
</tr>
<tr>
<td>(4,3)</td>
<td>329</td>
<td>10.018</td>
</tr>
<tr>
<td>(1,3)</td>
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<td>10.018</td>
</tr>
<tr>
<td>(3,1)</td>
<td>329</td>
<td>10.018</td>
</tr>
</tbody>
</table>
FIGURE 1 Model Structure

- Users’ trip chain and route choice behavior
- Origin
- Alternatives of Destination
- Destination
- Congestion in network
- Link travel time
- Link Traffic Flow
- Equilibrium
FIGURE 2 Test Network A
Note) Toll is charged only on link (1,0). Social surplus is normalized; surplus in no-toll equilibrium is 0% and that in optimal toll equilibrium is 100%.

FIGURE 3 Difference in Second-Best Toll between Trip-Based Model and Trip-Chain-Based Model
FIGURE 4 Test Network B
Note) Toll is charged only on links (1,2), (4,2), (3,2). Social surplus is normalized; surplus in no-toll equilibrium is 0% and that in optimal toll equilibrium is 100%.

FIGURE 5 Difference in Second-Best Cordon Toll between Trip-Based Model and Trip-Chain-Based Model